About the octuplet of symmetry-distinguishable direction indicators

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Physical quantities defined by a magnitude and an oriented axis in 3D space are often represented by three-component Euclidean vectors. Frequently, polar and axial (or pseudo-) vectors are distinguished, depending on whether they change their sense or not, respectively, upon the operation of spatial inversion (parity operation).[1–4] For classification of temporal processes or magnetic phenomena of vectorial nature, the action of the time-inversion operator (1′) can be used.

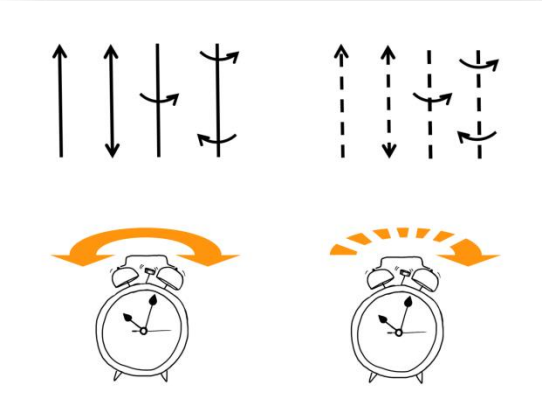

Figure 1 Pictograms providing intuitive definition of the octuplet of the symmetry-distinct vector-like quantities: vectors and bidirectors.

For example, magnetization M and magnetic field vector H are ”time-odd axial” vectors electric polarization P or electric field E are ”time-even polar” vectors, while other quantities like velocity v or toroidal moment T are ”time-odd polar” vectors.[1–6] The two inversion operations generate an Abelian (commutative) group of 4 elements with 4 one-dimensional irreducible representations; the symmetry operations this group allow to classify these vectors into 4 categories (see Table I).[1–4]

1	$\bar{1}$	1′	$\bar{1}'$	vectorial quantity	symbol
1	1	1	1	electric toroidal moment	**G**
1	-1	1	-1	electric dipole moment	**P**
1	1	-1	-1	magnetic dipole moment	**M**
1	-1	-1	1	(magnetic) toroidal moment	**T**

TABLE I: Action of space ($\bar{1}$) and time (1′) inversion operations on selected examples of vectorial quantities: 1 stands for the invariance, -1 stands for the sign-reversal.[3, 9, 12]

The aim of this contribution is to emphasize that there are another four types of quantities, which are also defined by a magnitude, an axis and a geometrical sign, and which are also often associated with three-component Euclidean vectors, but which possess a different spatiotemporal symmetry than the examples given in Table I): two kinds of chiral "bidirectors" C and F (associated with the so-called true and false chirality, resp.) and still another two "bidirectors" N and L, achiral ones, transforming as the nematic liquid crystal order parameter and as the antiferromagnetic order parameter of the hematite crystal α-Fe_2O_3, respectively. List of all kinds of the symmetry-distinct “vectorlike” quantities is given below and also illustrated in Fig. 1. Application of this classification to several selected issues discussed or cited in [7-13] will be given for illustration of the concept and the notation proposal.

		$\bar{1}$	$1'$	$m_{\parallel}$	limiting group
G	time-even axial	1	1	−1	$\infty/m.\mathbf{1}'$
P	time-even polar	−1	1	1	$\infty m.\mathbf{1}'$
M	time-odd axial	1	−1	−1	∞/mm'
T	time-odd polar	−1	−1	1	$\infty/m'm$
N	time-even neutral	1	1	1	$\infty/mm.\mathbf{1}'$
C	time-even chiral	−1	1	−1	$\infty 2.\mathbf{1}'$
L	time-odd neutral	1	−1	1	∞/mm
F	time-odd chiral	−1	−1	−1	$\infty/m'm'$

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